

Modeling of Near Wall Turbulence and Modeling of Bypass Transition

Z. Yang

1. Motivation and Objective

1) Modeling of the near wall turbulence. We aim to develop a second order closure for near wall turbulence. As a first step of this project, we try to develop a $k - \epsilon$ model for near wall turbulence. We require the resulting model to be able to handle both near wall turbulence and turbulent flows away from the wall, computationally robust, and applicable for complex flow situations, flow with separation, for example.

2) Modeling of the bypass transition. We aim to develop a bypass transition model which contains the effect of intermittency. Thus, the model can be used for both the transitional boundary layers and the turbulent boundary layers. We require the resulting model to give a good prediction of momentum and heat transfer within the transitional boundary and a good prediction of the effect of freestream turbulence on transitional boundary layers.

2. Work Accomplished

In the past year, progress has been made in both topics mentioned above (i.e., modeling of the near wall turbulence and modeling of the bypass transition). In the paragraphs below, these two topics will be reported separately.

2.1 Modeling of Near Wall Turbulence

Because of the wide range of scales involved in a turbulent flow, DNS (direct numerical simulation) is limited to flows of moderate Reynolds number and simple geometry. Turbulence modeling is the only viable approach for the calculation of turbulent flows of engineering interest. In turbulence modeling, the $k - \epsilon$ model is the most widely used model in engineering calculations. The Standard $k - \epsilon$ Model^{1,2} was devised for high Reynolds number turbulent flows and is traditionally used in conjunction with a wall function when it is applied to wall bounded turbulent flows. However universal wall functions do not exist in complex flows and it is thus necessary to develop a form of $k - \epsilon$ model equations which can be integrated down to the wall.

Jones and Launder³ were the first to propose a low Reynolds number $k - \epsilon$ model for near wall turbulence, which was then followed by a number of similar $k - \epsilon$ models. A critical evaluation of the pre-1985 models was made by Patel et al.⁴. More recently proposed models can be found in Shih⁵ and Lang and Shih⁶. Three major deficiencies can be pointed out about existing $k - \epsilon$ models. (Some of the models may have only one or two of the three deficiencies.) First, a near wall pseudo-dissipation rate was introduced to remove the singularity in the dissipation equation at the

wall. The definition of the near wall pseudo-dissipation rate was quite arbitrary. Second, the model constants were different from those of the Standard $k - \epsilon$ Model, making the near wall models less capable of handling flows containing both high Reynolds number turbulence and near wall turbulence, which often occurs for real flow situations. Patel et al.⁴ put as the first criterion the ability of the near wall models to predict turbulent free shear flows. Third, the variable y^+ is used in the damping function f_μ of the eddy viscosity formulae. Since the definition of y^+ involves u_τ , the friction velocity, any model containing y^+ cannot be used in flows with separation.

Effort is made to propose a new $k - \epsilon$ model for near turbulence which is free of the three deficiencies mentioned above. In this model, $k^{1/2}$ is chosen as the turbulent velocity scale. The time scale is bounded from below by the Kolmogorov time scale. The dissipation equation is reformulated using this time scale and no singularity exists at the wall. Thus, it is no longer necessary to introduce the near wall pseudo-dissipation rate. The model constants used are the same as in the Standard $k - \epsilon$ Model. Thus, the proposed model will be also suitable for flows away from the wall. An earlier version of the model, which contains y^+ in the damping function, was proposed and reported in Yang and Shih⁷. The model is now improved by using $R_y = k^{1/2}y/\nu$ instead of y^+ in the damping function. Hence, the present model can be used for flows with separation.

In the present model, the eddy viscosity is given by

$$\nu_T = c_\mu f_\mu k T \quad (1)$$

where the time scale T is written as

$$T = \frac{k}{\epsilon} + \left(\frac{\nu}{\epsilon}\right)^{1/2}. \quad (2)$$

The first part is the time scale conventionally used for high Reynolds number turbulent flows and the second part is the Kolmogorov time scale. Away from the wall, the first part is much large than the second part while near the wall the second part dominates, giving the Kolmogorov time scale as the turbulent time scale at the wall. The time scale given is bounded from below by the Kolmogorov time scale and is always positive.

The damping function f_μ is given by

$$f_\mu = [1 - \exp(-a_1 R_y - a_3 R_y^3 - a_5 R_y^5)]^{1/2} \quad (3)$$

where $a_1 = 1.5 \times 10^{-4}$, $a_3 = 5.0 \times 10^{-7}$, $a_5 = 1.0 \times 10^{-10}$. The damping function is chosen such that the shear stress has the correct near wall asymptotic behavior. Away from the wall, f_μ approaches one as required.

The modeled transport equations for k and ϵ are

$$\dot{k} + U_j k_{,j} = \left[\left(\nu + \frac{\nu_T}{\sigma_k}\right) k_{,j}\right]_{,j} - \langle u_i u_j \rangle U_{i,j} - \epsilon, \quad (4)$$

$$\dot{\epsilon} + U_j \epsilon_{,j} = \left[\left(\nu + \frac{\nu_T}{\sigma_\epsilon} \right) \epsilon_{,j} \right]_{,j} + (-C_{1\epsilon} \langle u_i u_j \rangle U_{i,j} - C_{2\epsilon} \epsilon) / T + \nu \nu_T U_{i,jk} U_{i,jk}. \quad (5)$$

Because T is always positive, the dissipation rate equation does not have a singularity at the wall.

Turbulent channel flows and flat plate boundary layer flows at different Reynolds numbers were calculated using the proposed model. The mean velocity, turbulent kinetic energy, turbulent shear stress and turbulent dissipation rate for turbulent channel flow at $Re_\tau = 395$ and turbulent flat plate boundary layer flow at $Re_\theta = 1410$ are shown in Fig 1 and Fig 2, respectively. DNS data for these cases are shown for comparison. Also shown are the predictions using the Jones-Launder model and the $k - \epsilon$ model proposed by Chien⁸. These two models are chosen because the Jones-Launder model is the first $k - \epsilon$ model for near wall turbulence while Chien's model is known to perform quite well for turbulent boundary layer flows. Overall, the proposed model is found to give a better prediction. Calculations were also made for turbulent flat plate boundary layers at larger Reynolds numbers, turbulent boundary layers with pressure gradient (favorable pressure gradient, adverse pressure gradient, and increasingly adverse pressure.) The results of these computation and the comparisons with the available experimental data can be found in Yang and Shih⁹.

2.2 Modeling of Bypass Transition

In a quiescent environment, transition is preceeded by the amplification of Tollmien-Schlichting waves. These waves eventually break down, giving rise to turbulent spots, which can be viewed as the onset of transition. In an environment with high freestream turbulence, say the flow passing over a turbine blade, turbulent spots are formed due to the transport of turbulence from the freestream to the boundary layer rather than the T-S wave amplification. This type of transition is called bypass transition. Accurate prediction of bypass transitional boundary layers is very important for internal fluid mechanics because a significant proportion of the turbine blade is in the a transitional boundary layer region. Furthermore, the performance and the life of a turbine are directly related to the peak vales of the momentum and heat transfer both of which occur in the transitional boundary layer.

Priddin¹⁰ was the first to notice that the low Reynolds number two equation models have the potential to predict transitional flows under the influence of the freestream turbulence. This is probably due to the fact that the generation of turbulent spots in a boundary layer is a random process and the flow is almost fully developed turbulent within a turbulent spot. A detailed calculation procedure was given by Rodi and Scheuerer¹¹, in which the Lam & Bremhorst low Reynolds number $k - \epsilon$ model was used. More recently, a comparative study of the performance of existing low Reynolds number $k - \epsilon$ models in predicting laminar-turbulent transition was made by Fujisawa¹².

While the low Reynolds number $k - \epsilon$ models could mimic transition, the quantitative predictions do not compare very well with the experimental data. This is due to the fact that all these low Reynolds number $k - \epsilon$ models were originally proposed for fully developed turbulent flows and did not take into consideration the

distinct feature of a transitional boundary layer – intermittency. The intermittency of a transitional boundary layer is measured by the intermittency factor which can be viewed as the percentage time a transitional boundary layer is in the turbulent state due to the passing of a turbulent spot.

We propose a model for the calculation of transitional boundary layers, which takes the effect of intermittency into consideration. The model is based on the $k - \epsilon$ model for near wall turbulence we have stated above. The effect of intermittency is introduced through the following argument: since the percentage of time that a transitional boundary layer is turbulent is measured by the intermittency factor, a model for transitional boundary layers could be constructed from a model for turbulent boundary layers by multiplying all the terms due to turbulence mechanism by a weighting factor γ , which is linearly related to the intermittency factor. Thus, the governing equations for the flat plate transitional boundary layers are, after using the boundary layer approximation and the eddy viscosity assumption,

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \quad (6)$$

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \frac{\partial}{\partial y} \left[(\nu + \gamma \nu_T) \frac{\partial U}{\partial y} \right], \quad (7)$$

$$U \frac{\partial k}{\partial x} + V \frac{\partial k}{\partial y} = \frac{\partial}{\partial y} \left[(\nu + \gamma \frac{\nu_T}{\sigma_k}) \frac{\partial k}{\partial y} \right] + \gamma \nu_T \left(\frac{\partial U}{\partial y} \right)^2 - \gamma \epsilon, \quad (8)$$

$$U \frac{\partial \epsilon}{\partial x} + V \frac{\partial \epsilon}{\partial y} = \frac{\partial}{\partial y} \left[(\nu + \gamma \frac{\nu_T}{\sigma_\epsilon}) \frac{\partial \epsilon}{\partial y} \right] + \gamma \left[C_{1\epsilon} \nu_T \left(\frac{\partial U}{\partial y} \right)^2 - C_{2\epsilon} \epsilon \right] \frac{1}{T} + \gamma \nu \nu_T \left(\frac{\partial U}{\partial y} \right)^2, \quad (9)$$

where x, y are the coordinates along and normal to the plate and U, V are the mean velocities in the x, y directions, respectively.

In order to close the above equations, an expression for the weighting factor γ is needed. The weighting factor is assumed to be related to both the freestream turbulent level and the intermittency factor of the boundary layer. The intermittency factor is assumed to be determined by the local state of the boundary layer. We use H , the shape factor, to characterize the local state of the boundary layer since both the intermittency factor and the shape factor change monotonically from the laminar boundary layer to the turbulent boundary layer. Experimental results by Abu-Ghannam and Shaw¹³ are used as a guide to construct this function. The weighting factor is also assumed to change in the y direction in such a way that outside the boundary layer, the weighting factor is one since the freestream is governed by decaying turbulence. The final expression for the weighting factor is given in Yang and Shih¹⁴.

The above system of parabolic equations need to be supplemented by boundary conditions at the wall and at the freestream and by initial conditions at the starting point of the calculation. At the wall

$$U = V = k = 0,$$

$$\epsilon_w = \frac{2}{\gamma} \nu \left(\frac{\partial k^{1/2}}{\partial y} \right)^2. \quad (10)$$

The boundary condition for ϵ was obtained by applying the equation for turbulent kinetic energy down to the wall.

At the edge of the boundary layer, the flow variables are given by the freestream values, i.e.

$$U = U_e, k = k_e, \epsilon = \epsilon_e. \quad (11)$$

The k_e, ϵ_e in the above are found from the transport equations for k and ϵ , with the condition that the gradient of the flow variables in the y direction vanishes as the free stream is approached. Thus,

$$U_e \frac{dk_e}{dx} = -\epsilon_e, \quad (12)$$

$$U_e \frac{d\epsilon_e}{dx} = -\frac{C_{2\epsilon}\epsilon_e}{T}. \quad (13)$$

k_{e0} and ϵ_{e0} (the values of k_e and ϵ_e at the leading edge, for example) are needed. k_{e0} is obtained from the experiment, and ϵ_{e0} is determined in such a way that the resulting $k_e(x)$ profile agrees with the experiment.

One of the issues in the calculation of transitional boundary layers through the low Reynolds number $k - \epsilon$ models is the prescription of the initial profiles for the turbulence kinetic energy and its dissipation rate, the later of which could not be found from the experiment directly. An expression for the initial profiles were given in Rodi and Scheuerer¹¹. However, computations by Yang and Shih¹⁵ which tested the effect of the initial conditions on the transition prediction found, in agreement with the findings of Patankar and Schmit¹⁶, that the predicted onset of the transition is sensitive to the initial profiles. This sensitivity of the results to the initial conditions suggests that the only place where the initial conditions could be specified unambiguously is at the leading edge. At the leading edge, the turbulent kinetic energy and its dissipation rate take constant profiles, the values of which are determined by the law for the decaying turbulence.

With the initial conditions given at the leading edge and the boundary conditions given above, the solutions are marched downstream. Flat plate boundary layers with free stream turbulence levels of 3% (Case T3A) and 6% (Case T3B) respectively were calculated using the present model. These are the benchmark cases in an ongoing project coordinated by Savill¹⁷, testing the capability of turbulence models in predicting transitional flows. Fig. 3 shows the variation of skin friction coefficient c_f against Re_x . Results from the experiment is shown for comparison. In addition, the prediction of the Launder-Sharma model is also shown in the figure because it was reported that among the lower Reynolds number $k - \epsilon$ models, the Launder-Sharma model performs best for transitional boundary layers. It is clear that the present model gives a better prediction. Other features in the transitional boundary layers and the calculations of the transitional boundary layers with other levels of freestream turbulence can be found in Ref. 14.

3. Future Plans

1) Modeling of the near wall turbulence.

The proposed $k - \epsilon$ model is only tested for simple parabolic flows so far. Because of the form of the model equations, the proposed model can be used in complex flow situations, flow with separation for example. The performance of the proposed model in those situations will be tested.

We will work on the second order closure for near wall turbulence. In particular, we will be looking at the effect of the mean flow inhomogeneity on the pressure strain correlation. We are hoping to represent this effect rationally, so that the ad hoc damping functions currently being used in all the near wall second order closures can be avoided.

2) Modeling of bypass transition.

We will apply the proposed model to transitional boundary layers with pressure gradient and curvature. We will also extend the model to thermal boundary layers.

4. References

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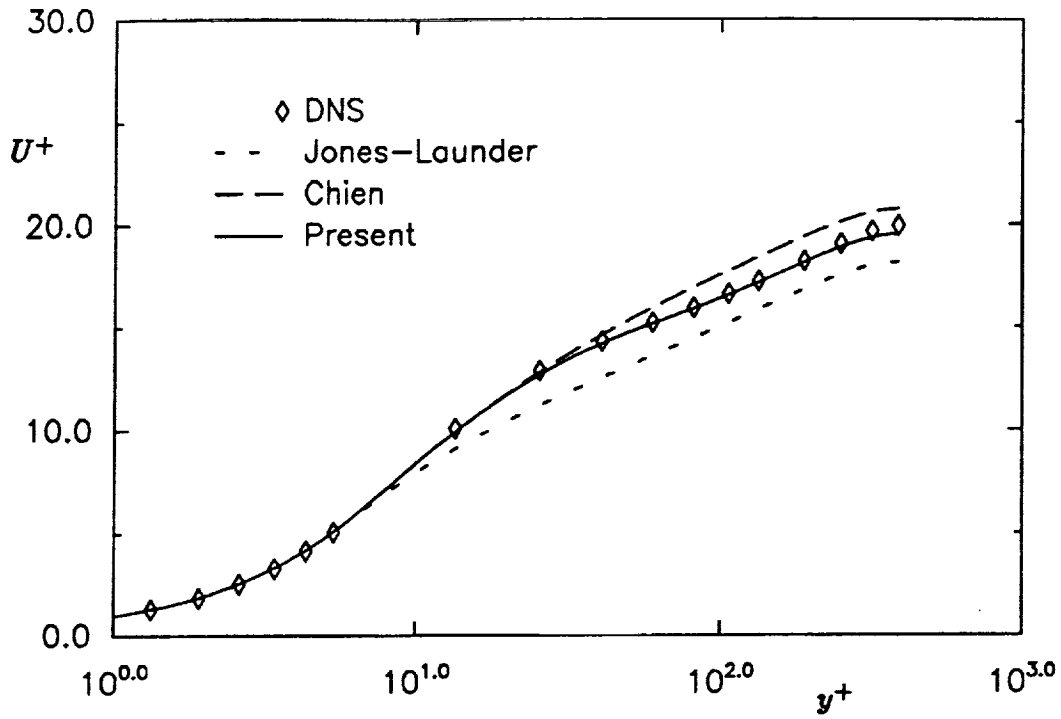


Fig 1a: Mean velocity profile for channel flow at $Re_\tau = 395$.

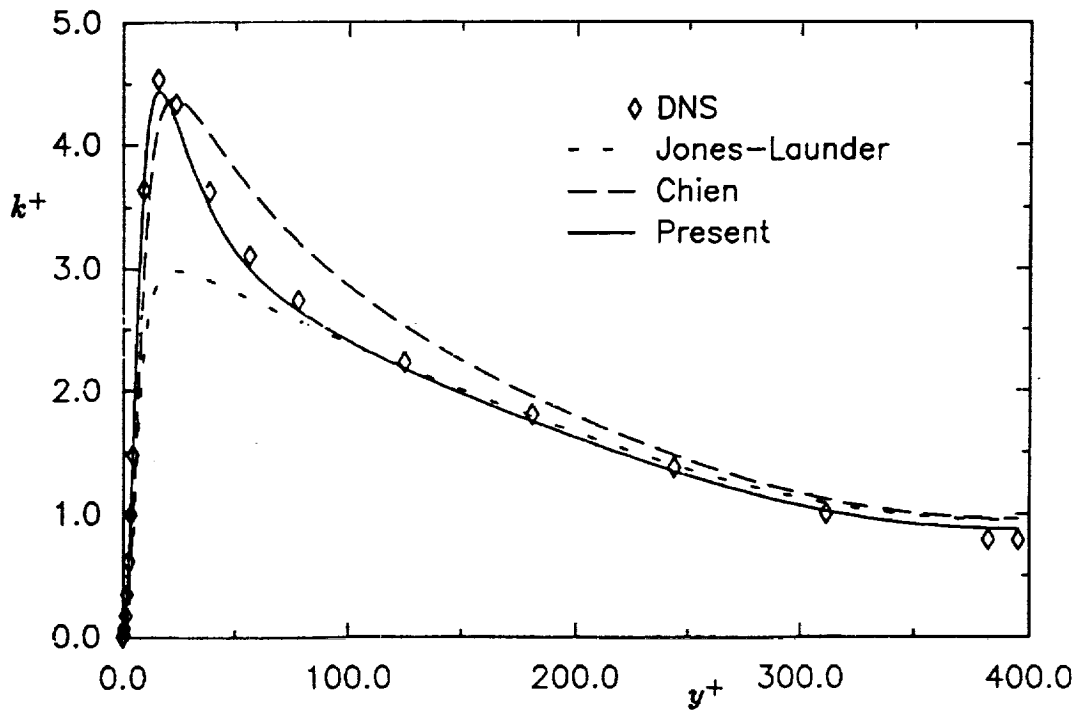


Fig 1b: Turbulent energy profile for channel flow at $Re_\tau = 395$.

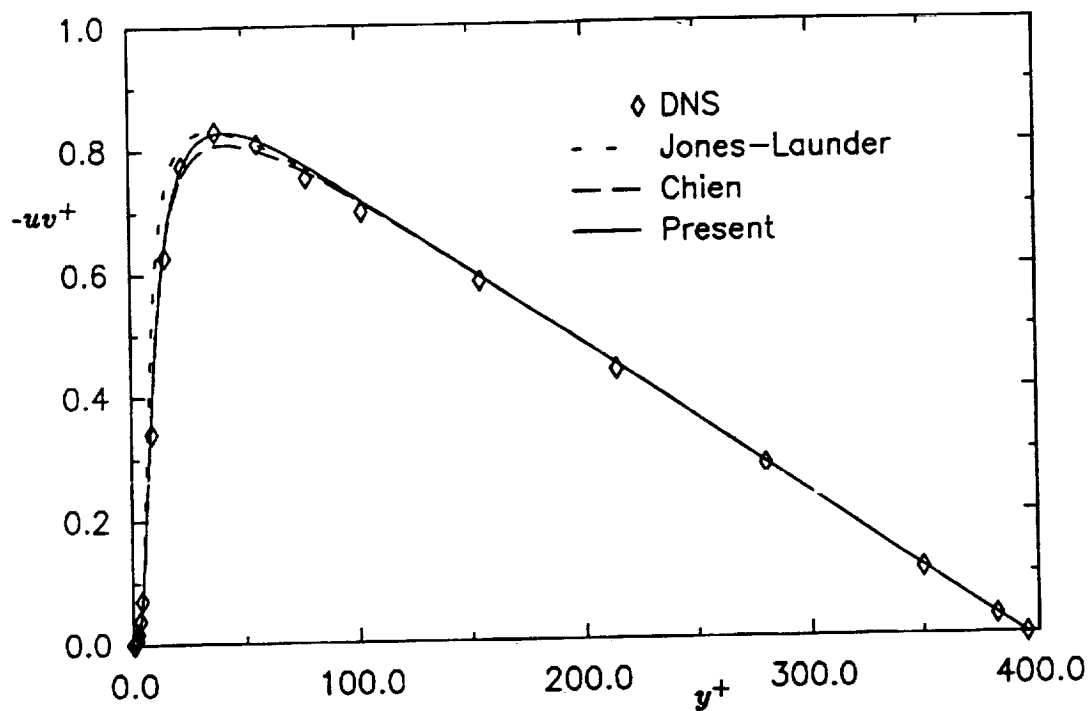


Fig 1c: Shear stress profile for channel flow at $Re_\tau = 395$.

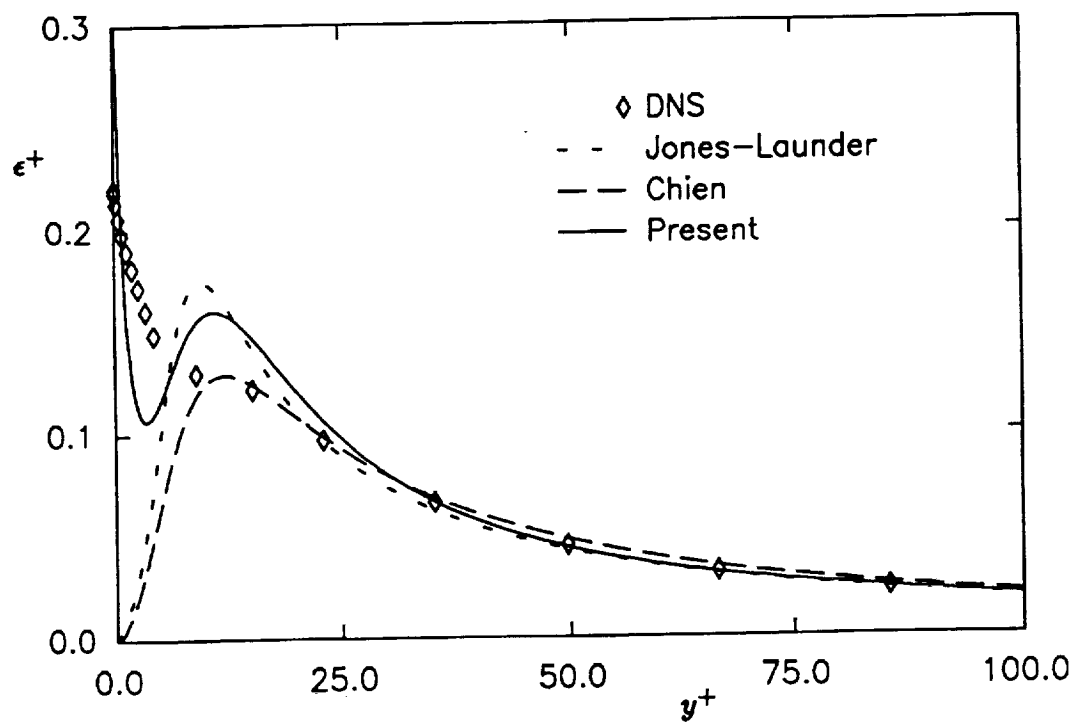


Fig 1d: Dissipation rate profile for channel flow at $Re_\tau = 395$.

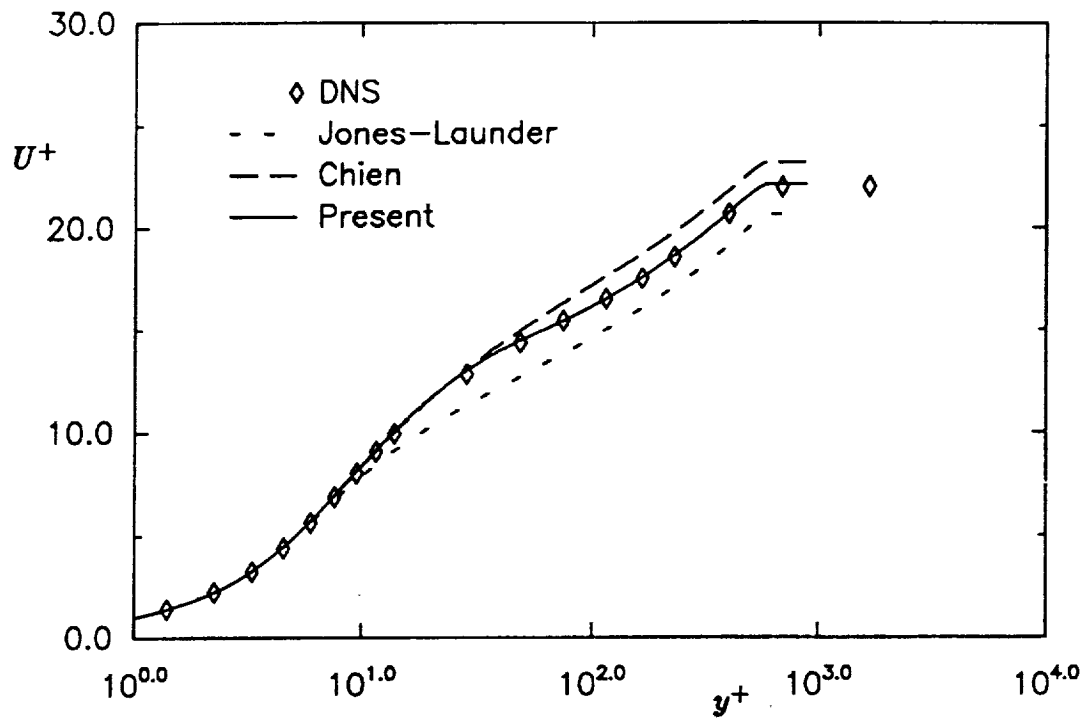


Fig 2a: Mean velocity profile for channel flow at $Re_\theta = 1410$.

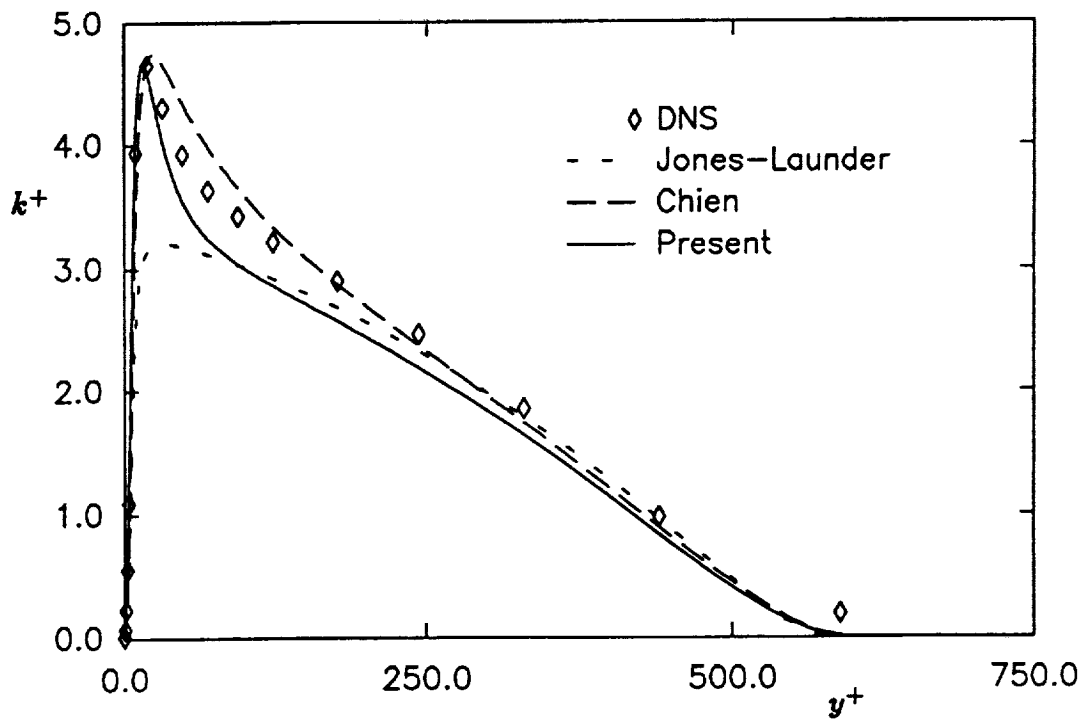


Fig 2b: Turbulent energy profile for channel flow at $Re_\theta = 1410$.

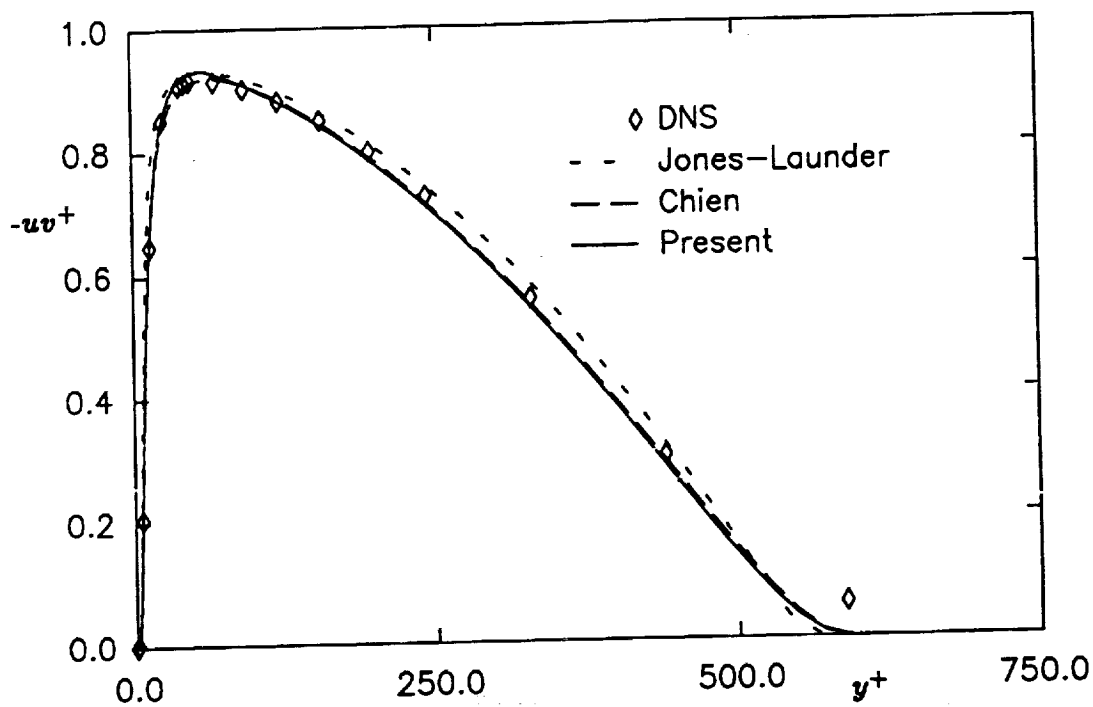


Fig 2c: Shear stress profile for channel flow at $Re_\theta = 1410$.

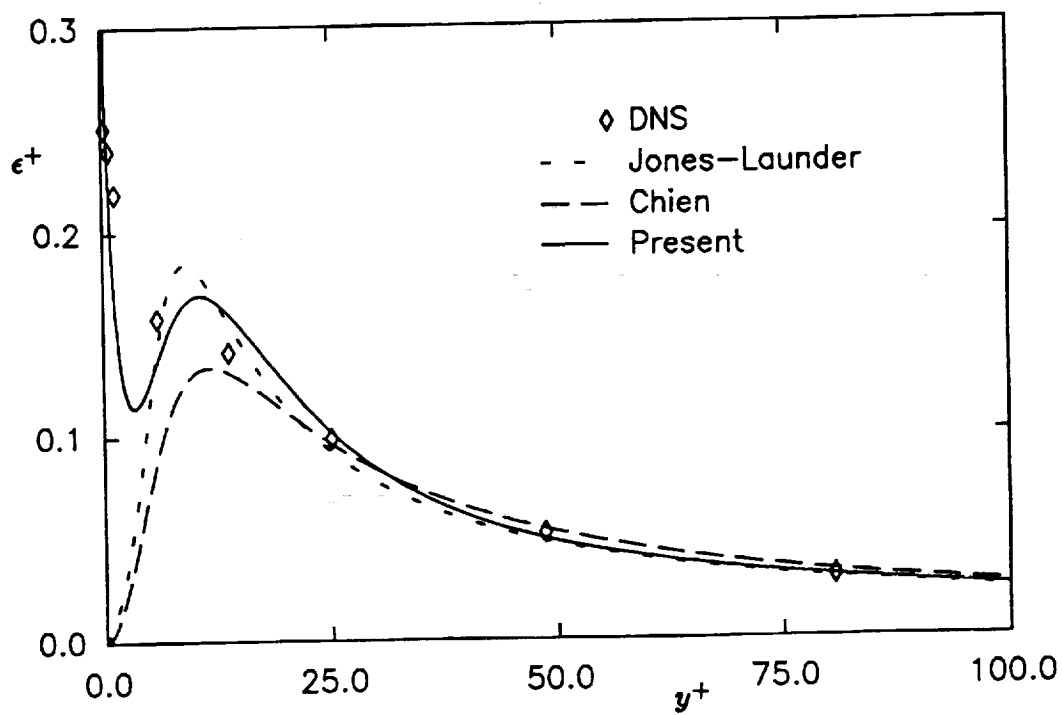


Fig 2d: Dissipation rate profile for channel flow at $Re_\theta = 1410$.

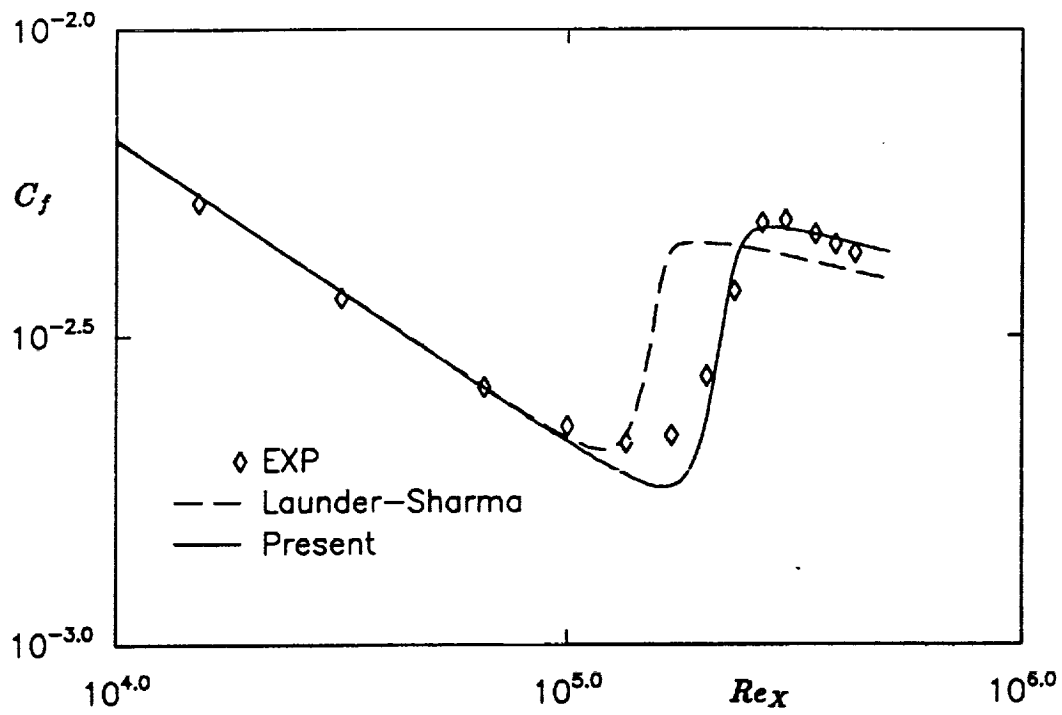


Fig 3a: Skin friction variation for T3A.

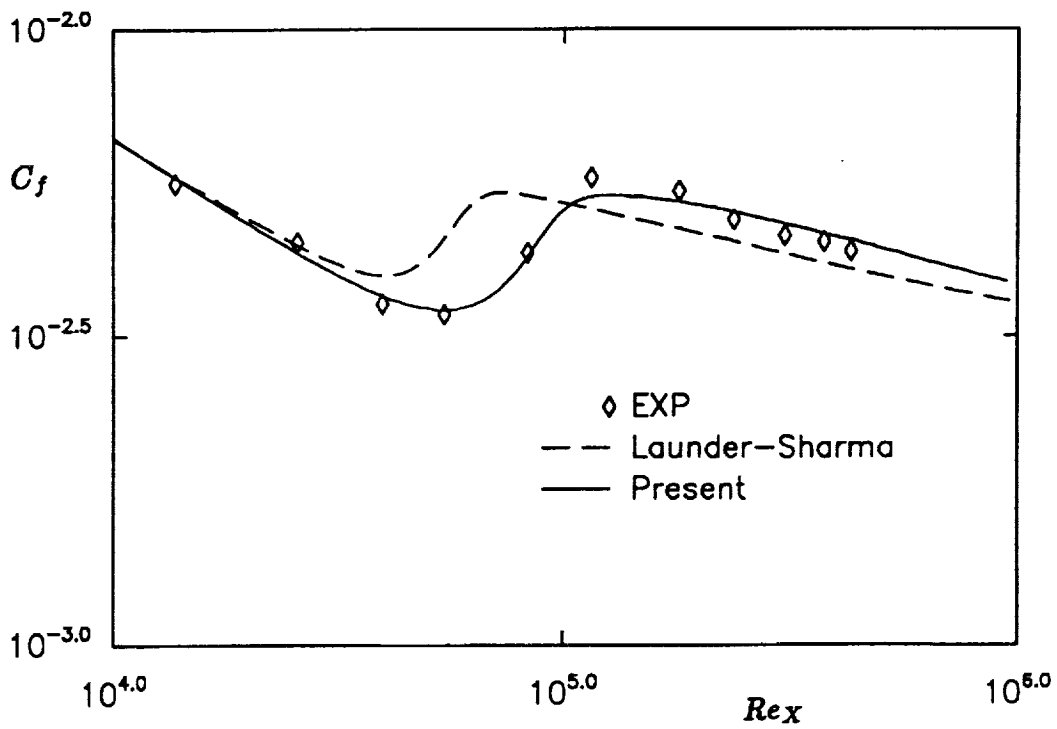


Fig 3b: Skin friction variation for T3B.